Parametric Amplification in Electrostrictively Doped Piezoelectric Semiconductors

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ABSTRACT
Using the coupled mode theory, a detailed analytical investigation of parametric amplification is made in a doped piezoelectric semiconductor employing the electrostrictive contribution of the crystal medium. The nonlinear induced current density and second-order effective optical susceptibility are obtained under off-resonant laser irradiation. The analysis deals with the qualitative behavior of the threshold value of pump electric field \( E_{0,\text{th}} \) for the onset of parametric amplification and parametric gain constant \( g_{\text{para}} \) with respect to excess doping concentration \( n_0 \) for different values of pump electric field. Numerical estimates are made for \( n \)-InSb crystal at liquid nitrogen temperature duly shined by a 10.6 \( \mu \)m CO\(_2\) laser. Efforts are directed towards optimizing the doping level and pump electric field to achieve maximum parametric amplification at pump fields much below the optical damage threshold. The analysis suggests that the threshold pump field required for the onset of parametric processes can be considerably reduced by selecting a moderately doped piezoelectric semiconductor with minimum electrostriction.

KEYWORDS
Parametric amplification, piezoelectric semiconductors, electrostriction, optical susceptibility.

INTRODUCTION
The phenomenon of parametric interaction (PI) exhibits a distinctive role in nonlinear optics. PI of a laser beam with a nonlinear medium results into generation of electromagnetic waves at new frequencies through mixing of the waves or controlled splitting which may undergo absorption/amplification depending on the material properties of the medium. Parametric processes have been widely used to generate tunable coherent radiation in a nonlinear medium at a frequency that is not directly available from a laser source [1, 2]. Parametric amplifiers (PA), parametric oscillators (PO), optical phase conjugation, pulse narrowing, squeezed state generation, etc. are some of the important devices and processes whose origin lies in PI in a nonlinear medium. Besides these technological uses, there are several other applications of PI in which researchers are interested [3, 4].

The developments in the field of PI in liquids and gases were thoroughly reviewed in full by Reintzes [5]. In solids, the initial theoretical review work has been done by Flytzanis [6]. Further developments in the field and the use of new wide frequency ranges for PO were severally restricted due to the transparency region of nonlinear medium. The doped inorganic semiconductors being transparent to photons of energy much below the forbidden energy of the material employed have proved themselves promising to yield sufficient parametric amplification [7].
Now a days, high mobility semiconductors have attracted much attention for their potential electronic and optical device applications due to their compactness, provision of control of material relaxation time and highly advanced fabrication technology. In addition, the large number of free electrons/holes available as majority charge carriers in doped semiconductors manifests many more exciting nonlinear optical processes [8]. Scattering of light beam from these free electrons in doped piezoelectric semiconductors were reported by Guha and Sen [9] and by Economou and Spector [10]. PI of acoustic waves with microwaves in piezoelectric semiconductors was studied by Economou and Spector [11]. PA of an acoustic wave in magnetized piezoelectric semiconductors have been studied analytically by Sharma and Ghosh [12]. The effects of carrier heating on PA in semiconductor-plasmas have been investigated by Singh et.al [13]. The threshold pump field and gain coefficient of PA in magnetoeactive III-V piezoelectric semiconductors have been analytically obtained by Lal and Aghamkar [14]. Recently, Choudhary et.al. [15] studied analytically the role of diffusion on parametric amplification/ attenuation of acoustic phonons in magnetized semiconductor plasmas.

Such critical analyses warrant the appropriate incorporation of various nonlinear processes with considerable impact on PIs at excitation intensities well above the threshold. One such important effect arises due to electrostriction. To the author’s knowledge, there exists no analytical result dealing with the parametric three-wave nonlinear optical processes in a doped piezoelectric semiconductor where electrostriction is not a trivial phenomenon.

In the present paper, the authors have analyzed theoretically following the coupled mode approach the effect of electrostriction on threshold behaviour and parametric gain in a doped piezoelectric semiconductor. It is found that the threshold pump field required for the onset of parametric processes can be minimized by selecting a semiconductor that demonstrates minimum electrostrictive effect.

We consider the bulk semiconductor crystal sample to be irradiated by a high power laser with photon energy (\(\hbar \omega_0\)) much below the forbidden energy gap (\(\hbar \omega_o\)) of the crystal. Numerical analysis is made for a representative weakly piezoelectric homogeneous III-V semiconductor with isotropic nondegenerate parabolic band structure, viz. n-InSb.

**THEORETICAL FORMULATIONS**

In order to study PA arising due to three wave mixing in an electrostrictively doped piezoelectric semiconductor duly shined by a relatively high power laser, we have derived analytically the expression for the complex effective second-order optical susceptibility (\(\chi^{(2)}\)). The three waves considered in the present scheme of parametric interaction are:

(i) the input spatially uniform (|\(k_0| \approx 0\)) pump electric field \(E = E_0 \exp(i \omega_0 t)\),

(ii) the idler acoustic phonon modes \(u(x,t) = u_n \exp[i(\omega_0 t - k_n x)]\), and

(iii) the scattered signal electromagnetic wave \(E_s(x,t) = E_s \exp[i(\omega_0 t - k_s x)]\).

For the sake of simplicity, in the forthcoming discussions, we write \(E_0(t) = E_o\) and \(E_s(x,t) = E_s\).

The momentum and energy exchange between these waves can be described by phase-matching conditions: \(\hbar k_s = \hbar k_0 + \hbar k_n\) and \(\hbar \omega_o = \hbar \omega_n + \hbar \omega_s\). In the interaction of high frequency electromagnetic waves and acoustic waves, it has been assumed without any loss of generality \(|k_o| (\approx k) \gg |k_n|\) under the dipole approximation.

In the present analysis, we use the well known hydrodynamic model of the homogeneous one component (viz. n-type) piezoelectric semiconductor crystal. The suitability of this model seems without any loss of significant information, by replacing the streaming electrons with an electron fluid described by a few macroscopic parameters like average carrier density, average velocity etc. However, this model restricts the analysis to be valid only in the limit \(k_o l \ll 1\) (\(k_o\) and \(l\) being the acoustic wave number and the electron mean free path, respectively).
The well established coupled mode approach is employed for the three wave mixing processes. The origin of PI lies in the coupling of pump and signal waves via density perturbations of the crystal medium.

The basic equations describing PI of the pump with the medium are as follows:

\[
\frac{\partial v_0}{\partial t} + v_0 v_0 = -\frac{e}{m} E_0
\]

(1)

\[
\frac{\partial v_1}{\partial t} + v_0 \frac{\partial v_1}{\partial x} + v_1 \frac{\partial v_1}{\partial x} = -\frac{e}{m} E_1
\]

(2)

\[
\frac{\partial n_0}{\partial t} + n_0 \frac{\partial n_0}{\partial x} + v_0 \frac{\partial n_0}{\partial x} = 0
\]

(3)

\[
\frac{\partial E_1}{\partial x} = -\frac{n_0 e}{\varepsilon} \frac{\partial^2 u}{\partial x^2} - \frac{\gamma n_0 e}{\varepsilon^2} \frac{\partial u}{\partial x}
\]

(4)

and

\[
\frac{\partial^2 u}{\partial t^2} = \frac{C}{\rho} \frac{\partial^2 u}{\partial x^2} - \frac{\beta}{\rho} \frac{\partial E_a}{\partial x} + \frac{\gamma (E_0 E_s)}{2\rho} \frac{\partial u}{\partial x} - 2\Gamma_a \frac{\partial u}{\partial t}
\]

(5)

Equations (1) and (2) are the zeroth and first-order momentum transfer equations in which \( v_0 \) and \( v_1 \) are, respectively, the zeroth and the first-order oscillatory fluid velocities of electrons of effective mass \( m \) and charge \(-e\), \( v \) being the electron collision frequency. Equation (3) is the continuity equation in which \( n_0 \) and \( n_1 \) are the equilibrium and perturbed electron densities, respectively. Equation (4) is the Poisson’s equation which determines the space charge field \( E_1 \), where \( \varepsilon \), \( \beta \) and \( \gamma \) are the dielectric constant, piezoelectric constant and electrostrictive coefficient of the crystal material, respectively. Equation (5) represents the motion of lattice vibrations in the crystal in which \( u \), \( \rho \), \( C \) and \( \Gamma_a \) are the relative displacement of oscillators from the mean position of the lattice, mass density of the crystal, elastic constant, linear elastic modulus of the crystal, and phenomenological damping parameter of acoustic mode, respectively. \( E_a \), \( E_0 \) and \( E_s \) are the electric fields associated with the acoustic vibration, pump electromagnetic wave and the Stoke’s mode, respectively.

It should be worth pointing out that, the coupling of electric and elastic properties of the lattice gives rise to the piezoelectric field \( E_a \) at the acoustic frequency \( \omega_a \). In a doped piezoelectric semiconductor, the low frequency generated acoustic wave \( (\omega_a) \) while interacting with high frequency pump electromagnetic wave \( (\omega_0) \) produces a.c. density perturbations \( (n_i) \) at frequencies \( \omega_0 \pm p\omega_a \), \( p \) being an integer, in the crystal medium. We are interested only in the lowest order with \( p = 1 \) representing the first-order Stoke’s component. Accordingly, the a.c. components such as \( v_1 \) and \( n_1 \) are assumed to oscillate at both the frequencies \( \omega_a \) as well as \( \omega_0 \). They are termed as the low and high frequency components, respectively. The perturbations at off-resonant frequencies are also neglected for \( p \geq 2 \).

Differentiating the continuity equation (3) and simplifying, we get

\[
\frac{\partial^2 n_1}{\partial t^2} + v \frac{\partial n_1}{\partial t} + \omega_p^2 k_s \left( \frac{k\beta}{e} n_i \gamma \frac{\partial n_1}{\partial x} \right) = -\frac{e}{m} \frac{\partial n_0}{\partial x} E_0,
\]

(6)

where \( \omega_p = \left( \frac{n_0 e^2}{mc} \right)^{1/2} \) is the electron plasma frequency of charge carriers.
In deriving Eq. (6), we have neglected the Doppler shift due to traveling space charge wave under the assumption $\omega_0 >> k_0 v_0$. Under rotating wave approximation (RWA), Eq. (6) gives two coupled equations for $n_{1s}$ and $n_{1a}$ as:

$$\frac{\partial^2 n_{1s}}{\partial t^2} + \nu \frac{\partial n_{1s}}{\partial t} + \omega_p^2 n_{1s} = -\frac{e}{m} \frac{\partial n_{1s}^*}{\partial x} E_0$$

(7a)

and

$$\frac{\partial^2 n_{1a}}{\partial t^2} + \nu \frac{\partial n_{1a}}{\partial t} + \omega_p^2 n_{1a} + \omega_p^2 k \left( \frac{n_{1p}}{\epsilon} - \frac{k \beta}{\epsilon} \right) = -\frac{e}{m} \frac{\partial n_{1a}^*}{\partial x} E_0.$$  

(7b)

Equations (7a) and (7b) exhibit the coupling between the low and high frequency components of the density perturbations $n_{1s}$ and $n_{1a}$ via the pump electric field $E_0$. Hence for the phenomenon of PI to occur, the presence of a pump field is the fundamental necessity.

Using Eqs. (5) and (7a,b) and making mathematical simplifications, we calculate the low- and high-frequency components of the density perturbations as:

$$n_{1s} = \frac{ie k E_0}{m(\omega_p^2 - \omega_s^2 - i\nu \omega_s)} n_{1s}^*$$

(8a)

and

$$n_{1a} = \frac{-ik^2 \omega_p^2 \left( \frac{n_{1p}}{\epsilon} - \frac{k \beta}{\epsilon} \right) \left[ E_{a\beta} - \frac{\gamma}{2} E_0 E_s^* \right]}{\left[ (\omega_p^2 - \omega_a^2 + i\nu \omega_a) + \frac{k^2 e^2 E_0^2}{m^2 (\omega_p^2 - \omega_s^2 - i\nu \omega_s)} \right] \rho (k^2 v_a^2 - \omega_a^2 - 2i\Gamma_a \omega_a)}$$

(8b)

where the term $(k^2 v_a^2 - \omega_a^2 - 2i\Gamma_a \omega_a)$ represents acoustic wave dispersion in the presence of damping. $v_a = (C / \rho)^{1/2}$ is the acoustic velocity in the crystal medium.

In order to study the role of electrostriction on the nonlinearity of the medium, we express the induced nonlinear current density $J_1(\omega_s)$ associated with the Stoke’s mode arising due to the coupling of the nonlinear carrier densities $n_{1s}$ and $n_{1a}$ expressed as:

$$J_1(\omega_s) = n_{1s} e v_{1s} + n_{1a}^* e v_{1a},$$

(9)

In the coupled-mode approach, the time integral of nonlinear current density $J_1(\omega_s)$ gives the effective complex induced polarization as:

$$P_{eff}(\omega_s) = \int J_1(\omega_s) dt = -\frac{i e}{\omega_s} (n_{1s}^* v_0 + n_{1a} v_{1a}).$$

(10)

The complex effective second-order optical susceptibility $\chi^{(2)}_{eff}$ can be obtained by defining the nonlinear polarization as:

$$P_{eff}(\omega_s) = v_0 \chi^{(2)}_{eff} E_0 E_s^*,$$

(11)

which gives

$$\chi^{(2)}_{eff} = \frac{\omega_p^2 k^2 e^2 \left( \frac{n_{1p}}{\epsilon} - \frac{ik \beta}{\epsilon} \right)}{m e_0 \rho \omega_0 (\omega_0 + i\nu) (k^2 v_a^2 - \omega_a^2 + 2i\Gamma_a \omega_a)} \left[ (\omega_p^2 - \omega_s^2 + i\nu \omega_s) + \frac{k^2 e^2 E_0^2}{m^2 (\omega_p^2 - \omega_s^2 - i\nu \omega_s)} \right].$$

(12)
We consider the semiconducting medium to be dispersionless for the acoustic waves (viz., $\omega_a = k v_a$) and the pump frequency $\omega_0$ considerably larger than the electron collision frequency $\nu$. For moderate doping, we can safely assume $\omega_0^2 << \omega_p^2$. Under these considerations, Eq. (12) reduces to:

$$\chi_{\text{eff}}^{(2)} = \frac{\omega_p^2 \Phi}{\omega_0 \omega_a \omega_s} \left( \frac{n_0 g}{\epsilon} + \frac{k \beta}{e} \right) \left[ \frac{(\overline{\omega}_s^2 + i \Omega_s)}{\nu_s^2 \omega_s^2 + k^2 e^2 E_0^2 + \omega_p^2 \omega_s^2 v^2} + i \nu (\overline{\omega}_s^2 \omega_s + \omega_p^2 \omega_s) \right]$$  \hspace{1cm} (13)

where $\Phi = \frac{k e^2 \beta}{2 m_e \rho v_a}$ and $\overline{\omega}_s^2 = \omega_s^2 - \omega_a^2$.

It is well known that PA can be achieved at pump fields above a certain threshold value. This threshold field can be obtained by considering Eq. (13) and proceeding as follows. First, we obtain the imaginary part of $\chi_{\text{eff}}^{(2)}$ from Eq. (13) using the relation $\chi_{\text{eff}}^{(2)} = \chi_{\text{Re,eff}}^{(2)} + i \chi_{\text{Im,eff}}^{(2)}$ as:

$$\chi_{\text{Im,eff}}^{(2)} = \frac{\omega_p^2 \Phi}{\omega_0 \omega_a \omega_s} \left( \frac{n_0 g}{\epsilon} \right) \left( \omega_a^4 \omega_s v^3 + \omega_a^4 \omega_s v^2 \right) - \left( \frac{k \beta}{e} \right) \left( \omega_a^2 \omega_s^2 v^3 - \omega_a^4 \omega_s \right) + \frac{k^2 e^2 E_0^2}{m^2} \left( \omega_a^2 \omega_s^2 v^2 + \omega_a^4 \omega_s \right) + \frac{k^2 e^2 E_0^2}{m^2} \left( \omega_a^2 \omega_s^2 v^2 + \omega_a^4 \omega_s \right) - \frac{k^2 e^2 E_0^2}{m^2} \left( \omega_a^2 \omega_s^2 v^2 + \omega_a^4 \omega_s \right) \left( \omega_a^2 \omega_s^2 v^2 + \omega_a^4 \omega_s \right)^2$$  \hspace{1cm} (14)

Equating $\chi_{\text{Im,eff}}^{(2)}$ to zero, the condition for the onset of the parametric process, the threshold value of the pump electric field can be obtained as:

$$|E_{0,\text{th}}^{\text{para}}|^2 = \frac{m}{ek} \left[ \frac{k \beta}{e} \left( \omega_a^2 \omega_s^2 v^3 + \omega_a^4 \omega_s v^2 \right) + \left( \frac{n_0 g}{\epsilon} \right) \left( \omega_a^2 \omega_s^2 v^3 + \omega_a^4 \omega_s v^2 \right) \right]^{1/2}.$$  \hspace{1cm} (15)

The parametric absorption/amplification coefficient $g_{\text{para}}$ of a parametrically excited wave-form of the pump field exceeding a threshold value is obtained through the relation [16]:

$$g_{\text{para}} = \frac{\omega_s}{\eta c} \chi_{\text{Im,eff}}^{(2)}$$

$$= \frac{\omega_p^2 \Phi}{\omega_0 \omega_a \eta c} \left( \frac{n_0 g}{\epsilon} \right) \left( \omega_a^4 \omega_s^2 v^3 + \omega_a^4 \omega_s^2 v^2 \right) - \left( \frac{k \beta}{e} \right) \left( \omega_a^2 \omega_s^2 v^3 - \omega_a^4 \omega_s \right) + \frac{k^2 e^2 E_0^2}{m^2} \left( \omega_a^2 \omega_s^2 v^2 + \omega_a^4 \omega_s \right) \left( \omega_a^2 \omega_s^2 v^2 + \omega_a^4 \omega_s \right)^2.$$  \hspace{1cm} (16)

where $\eta$ is background refractive index of crystal at frequency $\omega_0$ and $c$ is velocity of light.

The nonlinear parametric gain of the signal ($\omega_s$) as well as idler waves ($\omega_a$) can be possible only if $g_{\text{para}}$ obtained from Eq. (16) is negative for pump electric field $|E_0| > |E_{0,\text{th}}^{\text{para}}|$.
RESULTS AND DISCUSSION

We shall now address ourselves to a detailed numerical analysis of the threshold condition required for the onset of the parametric process and consequently the absorption/gain in an electrostrictively doped weakly piezoelectric semiconductors, viz., n-InSb crystal at liquid nitrogen temperature duly shined by nanosecond pulsed 10.6 μm CO₂ laser. The physical constants of n-InSb crystal are taken as follows: [15]:

\[ \rho = 5.8 \times 10^3 \text{ kg m}^{-3}, \quad m = 0.0145m_e, \quad m_e \text{ the free mass of electron}, \quad v = 3 \times 10^{11} \text{ s}^{-1}, \quad \varepsilon_i = 15.8, \quad \beta = 0.054 \text{ C m}^{-2}, \quad \gamma = 5 \times 10^{-10} \text{ SI units}, \quad \omega_a = 10^{12} \text{ s}^{-1}, \quad \nu_a = 4 \times 10^3 \text{ ms}^{-1}, \quad \omega_0 = 1.78 \times 10^{14} \text{ s}^{-1}, \quad \Gamma_a = 2 \times 10^{10} \text{ s}^{-1}, \quad \eta = 3.9. \]

Threshold Characteristics

Using Eq. (15) and the material parameters as given above, the detailed nature of dependence of the threshold pump field \( [E_{0,\text{th}}]_{\text{para}} \) on excess n-type doping concentration \( n_0 \) is investigated for n-InSb crystal and is plotted in Fig. 1. Curves (a) and (b) depict the features of \( [E_{0,\text{th}}]_{\text{para}} \) in the absence of electrostriction (\( \gamma = 0 \)) and in its presence (\( \gamma \neq 0 \)), respectively.

![Fig. 1. Variation of threshold electric field \( [E_{0,\text{th}}]_{\text{para}} \) of the parametric process with carrier concentration \( n_0 \) in n-InSb crystal. Curves (a) and (b) are for \( \gamma = 0 \) and \( \gamma \neq 0 \), respectively.](image)

It can be observed that for moderate doping region, \( [E_{0,\text{th}}]_{\text{para}} \) is found to behave almost identical with respect to \( n_0 \) except the important fact that a finite electrostrictive effect (\( \gamma \neq 0 \)) raises the threshold field by a factor of about 25 to 30 for \( n_0 = 5 \times 10^{23} \) and \( n_0 = 5 \times 10^{24} \text{ m}^{-3} \). Curve (a) in Fig. 1 depicts that in the absence of electrostrictive effect (\( \gamma = 0 \)), \( [E_{0,\text{th}}]_{\text{para}} \) decreases sharply with rising \( n_0 \), attaining a minimum value as
low as \( 3.3 \times 10^3 \) \( \text{V m}^{-1} \) at \( n_0 = 2.34 \times 10^{24} \) \( \text{m}^{-3} \). A further rise in \( n_0 \) increases the value of threshold electric field. For a semiconductor with finite electrostriction (\( \gamma \neq 0 \)), curve (b) in the same figure shows almost an identical response except that the fall in the threshold field value is not as large as in case of curve (a) with minimum value being \( 2.5 \times 10^6 \) \( \text{V m}^{-1} \) at \( 2.34 \times 10^{24} \) \( \text{m}^{-3} \).

**Parametric Absorption/Amplification**

We now concentrate ourselves to the quantitative analysis of the effective nonlinear parametric absorption coefficient \( g_{\text{para}} \) associated with the parametric excitation process in a piezoelectric semiconductor both in absence as well as presence of electrostriction (\( \gamma = 0 \) and \( \gamma \neq 0 \)) as a function of doping concentration \( n_0 \) and pump electric field \( E_0 \). We consider the pump field well above the threshold values (i.e., \( |E_0| > |E_{\text{th, para}}| \)) to achieve significant gain of PA.

![Graph showing the parametric absorption/amplification coefficient](image)

**Fig. 2.** Variation of parametric absorption/amplification coefficient \( \alpha_{\text{para}} \) at \( \gamma = 0 \) with carrier concentration \( n_0 \) in n-InSb crystal. Curves (a), (b), (c) and (d) are for \( E_0 = 10^5 \), \( 5 \times 10^5 \), \( 8 \times 10^5 \) and \( 10^6 \) \( \text{V m}^{-1} \), respectively.

Eq. (16) gives an expression for the parametric absorption/gain coefficient. It is well understood fact that a positive value of \( g_{\text{para}} \) gives parametric absorption while a negative value of \( g_{\text{para}} \) yields parametric gain or amplification. Usage of Eq. (16) will yield the information about parametric absorption/gain in the doped piezoelectric semiconductors.

Fig. (2) shows the nature of dependence of parametric absorption/gain coefficient \( g_{\text{para}} \) as a function of the excess electron concentration \( n_0 \) for different values of pump electric field strength \( E_0 \) in the piezoelectric...
crystal when electrostrictive effect is neglected \((\gamma = 0)\). We may notice from Fig. 2 that the magnitude of gain as well as the range of doping concentration at which gain occurs increases with increase of pump field strength \(E_0\). Moreover, for each value of \(E_0\), there exists a critical doping concentration \(n_0 = n_{0,cr1}\) above which \(g_{para}\) becomes negative and we achieve PA in the doped semiconductor. This amplification increases sharply with increase of \(n_0\) and becomes maximum at \(n_0 = 2.34 \times 10^{24} \text{ m}^{-3}\). If we further increase \(n_0\), gain coefficient starts reducing and beyond another critical value of \(n_0 = n_{0,cr2}\), gain disappears completely. The gain spectrum \(n_{0,cr1} \leq n_0 \leq n_{0,cr2}\) is larger at higher pump field strength. Thus, we may enhance significantly the PA by increasing slightly the pump field even for a fixed doping level in the semiconductor.

![Graph](image)

**Fig. 3. Variation of parametric absorption/amplification coefficient \(\alpha_{para}\) at \(\gamma \neq 0\) with carrier concentration \(n_0\) in n-InSb crystal. Curves (a), (b) and (c) are for \(E_0 = 8 \times 10^5\), \(1.5 \times 10^7\) and \(2 \times 10^7\) Vm\(^{-1}\), respectively.**

Fig. 3. shows the nature of dependence of parametric absorption/gain coefficient \(g_{para}\) on doping concentration \(n_0\) for three different values of pump electric field strength \(E_0\) in the piezoelectric crystal with finite electrostriction \((\gamma \neq 0)\). We may notice from Fig. 3 that the parametric absorption/gain feature in the electrostrictive case differs significantly from that observed for \(\gamma = 0\) in Fig. 2. The gain spectrum is much narrower and shifts to a smaller doping level with increase in the field strength. The interesting aspect of the parametric amplification in the case of \(\gamma \neq 0\) lies in its significant enhancement to about \(3 \times 10^{-3}\) V\(^{-1}\) at \(n_0 = 2 \times 10^{24} \text{ m}^{-3}\) and \(|E_0| = 2 \times 10^7\ \text{Vm}^{-1}\) from \(5 \times 10^{-10}\) in a medium with \(\gamma = 0\). From Fig. 3, it is also clear that an increase in field strength increases the gain regime. This feature is similar to that observed from Fig. 2 for the crystal with \(\gamma = 0\).
CONCLUSIONS
The present work deals with the analytical investigations of parametric amplification in an n-type doped weakly piezoelectric semiconductor viz., n-InSb duly shined by a nanosecond pulsed 10.6 μm CO₂ laser. The role of higher order forces such as electrostriction has been examined at length. The analysis enables one to draw the following conclusions:

1. The pump electric field produces a shift in the resonance frequency in second-order polarization term analogous to the so-called Stark shift and it plays an important role in the enhancement of parametric amplification.

2. The threshold pump field required for the onset of parametric excitation rises considerably in the presence of electrostriction. Hence, we should select a moderately doped n-type piezoelectric semiconductor for achieving reasonably high parametric amplification/gain.

REFERENCES